Math 10A with Professor Stankova
Quiz 4; Wednesday, 9/20/2017
Section #106; Time: 10 AM
GSI name: Roy Zhao **Name:** _____

Circle True or False or leave blank. (1 point for correct answer, -1 for incorrect answer, 0 if left blank)

1. True **FALSE** If a function f has a local maximum at $x = c$, then $f''(c) < 0$.

2. True **FALSE** For a function $f : [a, b] \to \mathbb{R}$, the set of critical points of f is $\{x \in [a, b] : f'(x) = 0\}$.

Show your work and justify your answers. Please include all units in the final answer.

3. (10 points) Oski is $1m$ tall and standing $100m$ away from the base of a $100m$ pole. On top of this pole is a set of floodlights which are shining down on him.

   (a) (6 points) Write a formula that expresses the height of Oski's shadow x as a function of how high the floodlights are h. (Write a formula only involving x, h, and constants).

   Solution: Draw a line from the floodlights to the top of Oski's head. The height of Oski's shadow is the distance from Oski's feet and where this line intersects the ground. Using similar triangles, the ratio between Oski's height and the height of the shadow is equal to the ratio between the height of the floodlights and distance the base of the pole is to the intersection point. So written out, it says
   $$\frac{1}{x} = \frac{h}{x + 100}.$$
   Clearing denominators gives $x + 100 = hx$. We can also write this as $(h - 1)x = 100$.

(b) (2 points) The floodlight suddenly falls and is falling at a constant rate of $1m/s$. How fast is the length of Oski's shadow changing when the floodlight is $51m$ from the ground? Interpret your answer (lengthening vs. shortening).

Solution: We use the formula to calculate when $h = 51$, then $50x = 100$ so $x = 2m$. Taking the derivative of our formula from above with respect to t gives

$$(h-1)\frac{dx}{dt} + x\frac{dh}{dt} = 0.$$

Now we plug in -1 for h' since the light is falling, and 51 for h and $2m$ for x to get

$$50\frac{dx}{dt} + 2(-1) = 0 \implies \frac{dx}{dt} = \frac{2}{50} = \frac{1}{25} = 0.04m/s.$$

His shadow is lengthening at a rate of $4cm/s$.

(c) (2 points) The floodlight is now falling at a constant rate of $2m/s$. How fast is the length of Oski's shadow changing when the floodlight is $51m$ from the ground? Interpret your answer (lengthening vs. shortening).

Solution: We can do the same calculation as above and now we plug in -2 for h' to get that

$$50\frac{dx}{dt} + 2(-2) = 0 \implies \frac{dx}{dt} = \frac{4}{50} = 0.08m/s.$$

His shadow is lengthening at a rate of $8cm/s$.